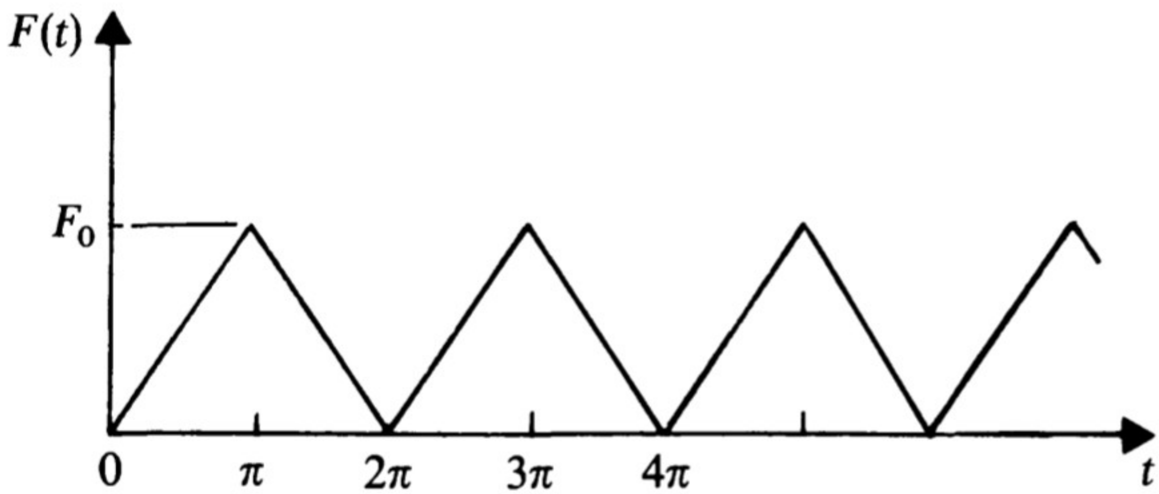
Practice Problem Set 5A

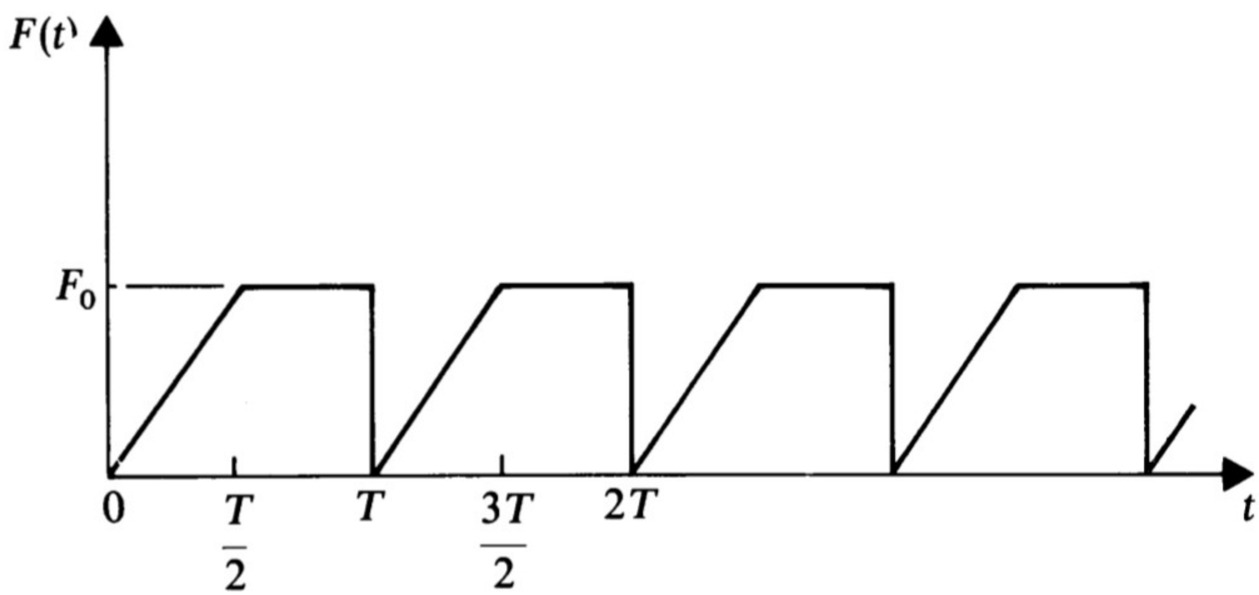
Fourier Series Approx of Periodic Forcing Function

Do the following problems from the book, <u>then plot the Fourier series</u>.

5.6: Find the Fourier series expansion of the function shown below:



5.13: Find the Fourier series expansion of the function shown below:



SCROLL

DOWN

FOR

SOLUTION

(But don't get tempted by the dark side. Resist! Use the, um, Force?)

ARE

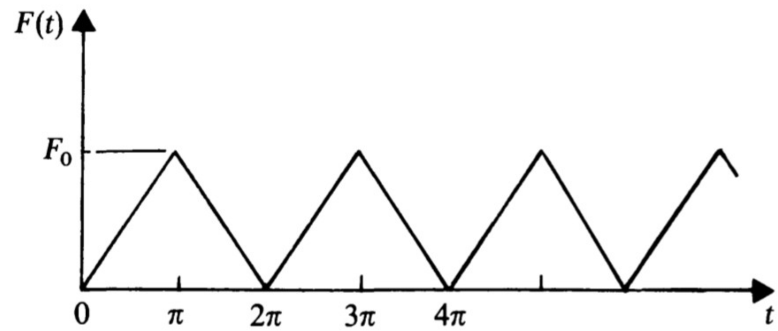
YOU

SURE?

(Go back up and think harder? Also, what exactly are you looking for in the solution below?)

SOLUTION

5.6: Find the Fourier series expansion of the function shown below:



(6)

$$F(t) = \begin{cases} \frac{F_0}{\pi} t & 0 \le t \le \pi \\ -\frac{F_0}{\pi} t & -\pi \le t \le 0 \end{cases}$$

$$T_f = 2\pi \quad , \quad \omega_f = \frac{2\pi}{T_f} = 1$$

$$F(t) = F(-t)$$

$\Rightarrow$ F(t) is an even function

$$b_n = 0$$

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_f t$$

$$a_0 = \frac{2}{T_f} \int_{-\frac{T_f}{2}}^{\frac{T_f}{2}} F(t)\, dt$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} \frac{(-F_0)}{\pi} t\, dt + \int_0^{\pi} \frac{F_0}{\pi} t\, dt \right.$$

$$= F_0$$

(6) Cont'd

$$a_n = \frac{2}{T_f} \int_{-\frac{T_f}{2}}^{\frac{T_f}{2}} F(t) \cos n\omega_f t\, dt$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} \frac{(-F_0)}{\pi} t \cos nt\, dt + \int_0^{\pi} \frac{F_0}{\pi} t \cos nt\, dt \right.$$

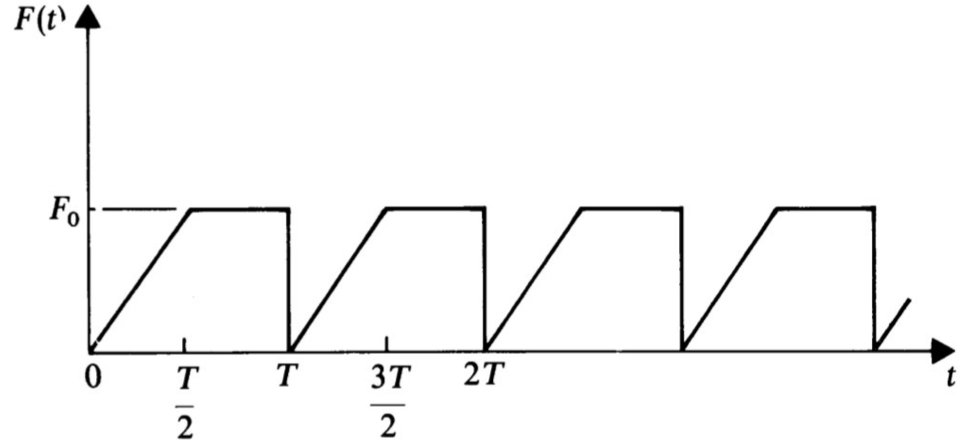
$$= \frac{2F_0}{(n\pi)^2} \left[(-1)^n - 1 \right]$$

$$\therefore \; a_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{-4F_0}{(n\pi)^2} & \text{if } n \text{ is odd} \end{cases}$$

Therefore

$$F(t) = \frac{F_0}{2} - \sum_{n=1,3,5}^{\infty} \frac{4F_0}{(n\pi)^2} \cos nt$$

5.13: Find the Fourier series expansion of the function shown below:



(13)

$$F(t) = \begin{cases} \frac{2F_o}{T} t & 0 \le t \le \frac{T}{2} \\ F_o & -\frac{T}{2} \le t \le 0 \end{cases}$$

$$T_f = T \ , \quad \omega_f = \frac{2\pi}{T_f} = \frac{2\pi}{T}$$

$$F(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_f t + b_n \sin n\omega_f t \right)$$

$$a_o = \frac{2}{T_f} \int_{-\frac{T_f}{2}}^{\frac{T_f}{2}} F(t)\, dt$$

$$= \frac{2}{T} \left(\int_{-\frac{T}{2}}^{0} F_o\, dt + \int_{0}^{\frac{T}{2}} \frac{2F_o}{T} t\, dt \right)$$

$$= \frac{3}{2} F_o$$

$$a_n = \frac{2}{T_f} \int_{-\frac{T_f}{2}}^{\frac{T_f}{2}} F(t) \cos n\omega_f t\, dt$$

$$= \frac{2}{T} \left(\int_{-\frac{T}{2}}^{0} F_o \cos \frac{2n\pi}{T} t\, dt + \int_{0}^{\frac{T}{2}} \frac{2F_o}{T} t \cos \frac{2n\pi}{T} t\, dt \right)$$

$$= \frac{F_o}{(n\pi)^2} \left[(-1)^n - 1 \right]$$

$$a_n = \begin{cases} \frac{-2F_o}{(n\pi)^2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

(13) Cont'd

$$b_n = \frac{2}{T_f} \int_{-\frac{T_f}{2}}^{\frac{T_f}{2}} F(t) \sin n\omega_f t\, dt$$

$$= \frac{2}{T} \left(\int_{-\frac{T}{2}}^{0} F_o \sin \frac{2n\pi}{T} t\, dt + \int_{0}^{\frac{T}{2}} \frac{2F_o}{T} t \sin \frac{2n\pi}{T} t\, dt \right)$$

$$= -\frac{F_o}{n\pi}$$

$$\therefore \ F(t) = \frac{3}{4} F_o - \sum_{n=1,3,5}^{\infty} \frac{2F_o}{(n\pi)^2} \cos \frac{2n\pi}{T} t - \sum_{n=1}^{\infty} \frac{F_o}{n\pi} \sin \frac{2n\pi}{T} t$$